Experimental Verification of the Relationship between Diffusion Constant and Mobility of Electrons and Holes

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The relationship between diffusion constant and mobility, called the Einstein relationship, has been experimentally verified for electrons and holes in germanium. This has been accomplished by measuring the rate of increase in half concentration width of a pulse of minority carriers moving in an electric field.

ACCORDING to theory, the ratio of the diffusion constant D to the mobility μ of charged particles is

$$D/\mu=kT/q,$$

where T is the absolute temperature, k the Boltzmann constant, and q is the magnitude of the charge.[1]

This relation was verified by the following experiment. Sharp pulses of minority carriers were injected into single crystal rods of both P- and N-type germanium in which there was an electric field. As the pulse moved along the rod under the influence of the electric field, diffusion occurred, resulting in a continuous transition from a sharp to a broad pulse. The essential features of the pulse shape were determined at two different distances along the rod.

The pulse was injected at an emitter point and detected at a collector point, which was placed some distance away. The variation of current through the collector point was observed with an oscilloscope, using essentially the same arrangement employed by Haynes and Westphal in the determination of the mobility of holes and electrons in silicon.[2]

It can be shown that to a good approximation

$$\frac{D}{\mu}=\frac{V(\Delta t_1^2-\Delta t_2^2)}{11.08t_1(t_1-t_2)}$$

where Δt_1 = width of the pulse at half-maximum corresponding to the transit time t_1, Δt_2 = width of the pulse at half-maximum corresponding to the transit time t_2, and V = potential difference between emitter and collector positions for transit time t_1.

Using a constant electric field, the transit times and pulse widths were measured on the time base of the cathode-ray oscilloscope for two positions of the emitter point.

Since the determination of D/μ depends on a difference of squares of measured quantities, single observations do not have a high level of significance. Independent measurements were made by a large number of observers. The average value for D/μ obtained using injected electrons is within 1 percent of that obtained with holes. Since analysis shows that the data have a normal probability distribution, the method of least squares was used to evaluate the most probable value and its significance. The following results were

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[1] This equation is known as the Einstein relationship [see, for instance, N. F. Mott and R. W. Gurney, *Electronic Processes in Ionic Crystals* (Clarendon Press, Oxford, 1940), p. 63, or W. Shockley, *Electrons and Holes in Semiconductors* (D. Van Nostrand Company, Inc., New York, 1950), p. 300]. As pointed out by E. Spenke of Siemens-Schuckertwerke, however (private communication to W. Shockley), the essentials of this equation, published by A. Einstein [Ann. Physik **17**, 549 (1905)] had been published previously by J. S. Townsend [Trans. Roy. Soc. (London) **A193**, 129 (1900)] and by W. Nernst [Z. Physik. Chem. **9**, 613 (1884)]. Possibly it should be called the Nernst-Townsend-Einstein relationship since each derivation appears independent of the others.

[2] J. R. Haynes and W. C. Westphal, Phys. Rev. **85**, 680 (1952).

obtained:[3]

$$D/\mu=0.0268\pm0.0013 \text{ ev},$$
$$T=303\pm1°\text{K},$$
$$kT/q=0.0262\pm0.0001 \text{ ev}.$$

[3] The probable error in the values of T and kT/q were not obtained using the method of least squares which would give a value much less than this. Generous allowance is made for systematic uncertainty in recording ambient temperature.

It would appear that these results verify the relation $D/\mu=kT/q$. Although there have been other experimental verifications of this relationship using colloidal particles and ions, this is the first direct experimental proof of the validity of this equation for electrons and holes of which we are aware.

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In this paper Einstein's unified field theory is modified, and some of the physical implications of the new theory are examined. It entails: (1) a restriction of 4-current distribution, (2) an electromagnetic field consisting of short- and long-range parts, (3) a finite self-energy for the electron, (4) a classical description of pair production and annihilation as discussed by Feynman in his electrodynamics, (5) the Lorentz-force law for a charged particle moving in an external electromagnetic field, (6) the bending of light grazing the surface of the sun—the same as given by the general theory of relativity.

1. INTRODUCTION

THE arguments for the necessity of a unified field theory are well known, and therefore they will not be elaborated at length. The author believes that a correct and unified quantum theory of fields, without the need of the so-called renormalization of some physical constants, can be reached only through a complete classical field theory that does not exclude gravitational phenomena. It is true that one cannot feel very optimistic about the quantization of a nonlinear classical field theory. But one hopes that this difficulty may be overcome, partly, by starting the quantization procedure with a Lagrangian[1] formulation of the quantum field theory.

In this paper we propose a new version of Einstein's latest unified field theory.[1a] The reasons for this modification will be made clear in the following. The same formalism and notation of Einstein's theory are used. The total field is described by a Hermitian tensor $g_{\alpha\beta}$ given as

$$g_{\alpha\beta}=a_{\alpha\beta}+i\varphi_{\alpha\beta}, \tag{1.1}$$

where

$$a_{\alpha\beta}=g_{\underline{\alpha\beta}} \quad \text{and} \quad \varphi_{\alpha\beta}=g_{\underset{\smile}{\alpha\beta}}, \quad i=(-1)^{\frac{1}{2}},$$

so that we have

$$(g_{\alpha\beta})^{\dagger}=(g_{\alpha\beta}). \tag{1.2}$$

The dagger (†) stands for Hermitian conjugate operation. We also have the general affine connection $\Gamma^{\gamma}_{\alpha\beta}$ given by

$$\Gamma^{\gamma}_{\alpha\beta}=\Gamma^{\gamma}_{\underline{\alpha\beta}}+i\Gamma^{\gamma}_{\underset{\smile}{\alpha\beta}}. \tag{1.3}$$

The Hermitian property of $\Gamma^{\gamma}_{\alpha\beta}$ in the covariant indices α and β is obvious.

Now, if we define $a^{\alpha\beta}$ as the normalized minors of $\text{Det}a_{\alpha\beta}=a$, then, we have

$$a_{\alpha\mu}a^{\gamma\mu}=\delta_{\alpha}{}^{\gamma}.$$

The determinant of $g_{\alpha\beta}$, because of (1.2), is real and can be expressed as

$$g=a(1-\Omega-\Lambda^2), \tag{1.4}$$

where

$$\Omega=\tfrac{1}{2}\varphi_{\mu\nu}\varphi^{\mu\nu} \quad \text{(is an invariant)},$$
$$\Lambda=\tfrac{1}{4}f^{\mu\nu}\varphi_{\mu\nu} \quad \text{(is a pseudoscalar)},$$

and

$$f^{\alpha\beta}=\frac{1}{2(-a)^{\frac{1}{2}}}\epsilon^{\alpha\beta\mu\nu}\varphi_{\mu\nu}, \tag{1.5}$$

where $\epsilon^{\alpha\beta\mu\nu}$ is zero whenever any two indices are equal and is ±1 for even and odd permutations. All indices are raised by $a^{\alpha\beta}$.

We also have the contravariant tensor $g^{\alpha\beta}$ given by

$$g_{\alpha\mu}g^{\beta\mu}=\delta_{\alpha}{}^{\beta}.$$

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[1] J. Schwinger, Phys. Rev. 82, 914 (1951).
[1a] A. Einstein, *The Meaning of Relativity* (Methuen, London, 1951).